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DICTIONARY APPROACHES TO IMAGE COMPRESSION AND RECONSTRUCTION

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ABSTRACT

This paper proposes using a collection of parameterized waveforms, known as a dictionary, for the purpose of medical image compression. These waveforms, denoted as φ_γ, are discrete time signals, where γ represents the dictionary index. A dictionary with a collection of these waveforms is typically complete or overcomplete. Given such a dictionary, the goal is to obtain a representation image based on the dictionary. We examine the effectiveness of applying Basis Pursuit (BP), Best Orthogonal Basis (BOB), Matching Pursuits (MP), and the Method of Frames (MOF) methods for the compression of digitized radiological images with a wavelet-packet dictionary. The performance of these algorithms is studied for medical images with and without additive noise.

Keywords: Image coding and compression, Medical Image Processing, Signal Reconstruction, Wavelets

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1 INTRODUCTION

We will discuss the advantages and disadvantages of using four methods of decomposition for image compression and restoration. The methods are Method of Frames (MOF) [1], Best Orthogonal Basis (BOB) [3], Matching Pursuit (MP) [4], and Basis Pursuit (BP) [2]. What these methods have in common is a requirement to use waveforms from a "dictionary" to represent an image. A dictionary, Φ, is simply a collection of parameterized waveforms, φ_γ, used as a basis for analysis. The parameter γ is dependent upon the dictionary type, e.g. if using a frequency dictionary, then γ is the indexing frequency. We are interested in these methods because they offer a flexible mechanism to customize a dictionary with known waveforms. This would allow higher compression of images using a customized dictionary.

2 METHOD OF FRAMES

Given a discrete dictionary of p waveforms (each of length n) that are collected as columns of an $n \times p$ matrix, Φ, the decomposition problem is:

$$\Phi\alpha = f. \qquad (1)$$

The Method of Frames (MOF) uses either a wavelet packet or cosine packet dictionary to pick out among all solutions of equation (1), the solution whose coefficients have the minimum L^2 norm:

$$\min \|\alpha\|_2 \quad \text{subject to } \Phi\alpha = f \qquad (2)$$

The MOF solution is obtained by the use of a conjugate gradient method [5] to solve the equation.

There are two key limitations with the Method of Frames (MOF). The first is that MOF is not sparsity-preserving. MOF tends to use all the basis functions nonorthogonal to the signal yielding a very non-sparse representation. If the signal can be represented by a minimal set of the dictionary, then the coefficients found by MOF are likely to be more than this minimal set. The second limitation is that the MOF is resolution-limited. Specifically, no object can be reconstructed with features sharper than those allowed by the analysis and synthesis operators. This has been shown in [2].

3 BEST ORTHOGONAL BASIS

The Best Orthogonal Basis (BOB) method originated by Coifman and Wickerhauser [3,6] seeks to find a best basis out of an orthogonal set of vectors relative to a given signal. Thus, overall information cost is optimized. This method uses a library of orthogonal waveforms that has a natural dyadic tree structure. Utilizing this type of structured dictionary makes it easy to construct orthogonal bases by an O(N log N) search algorithm.

Given a library as a tree structure, the best basis of a signal *f* is found by traversing the tree and selecting nodes that correspond to a minimization of the entropy function. The union of these nodes correspond to the best basis [3]. Shannon's entropy function is used as the selection criteria

4 MATCHING PURSUIT

Mallat et. al. [4] has introduced an algorithm that can provide a decomposition of signals that vary widely in both time and frequency. It decomposes any signal into a linear expansion of waveforms that are selected from a list or dictionary of functions. It chooses a waveform that best matches the signal structure of the signal at each iteration. The remaining portion that is unmatched reprocessed at the next iteration and matched to another signal in the dictionary. This process continues until a specified error tolerance is reached.

This algorithm can be expressed as a simple decomposition by inner product of dictionary elements on successive residuals.

$$f = \sum_{n=0}^{m-1} \langle R^n, \varphi_{\gamma_n} \rangle \varphi_{\gamma_n} + R^m \qquad (3)$$

R^m is the residue vector after approximating f at the mth iteration. In this algorithm, one begins by computing the inner products in a dictionary. The elements of the dictionary are chosen in a way such that

$$\left| \langle R^n, \varphi_{\tilde{\gamma}} \rangle \right| = \sup_{\varphi_\gamma \in \varphi} \left| \langle R^n, \varphi_\gamma \rangle \right| \qquad (4)$$

i.e. find the φ_γ that produces the maximum inner product.

The Matching Pursuit algorithm is greedy. This means that it must compute all the inner products within the dictionary to compute its solution. As a result, this method will take longer to compute in overcomplete dictionaries because it must first make a calculation for an atom that would be the best fit on the data. After this initial guess, the residue function could turn out to be more complex and the MP algorithm continues in a fashion to correct the errors from initial guess. This will result in sub-optimal fitting of the other terms in the decomposition. It will however do well with orthogonal dictionaries

5 BASIS PURSUIT

Basis Pursuit (BP) determines a signal representation such that the coefficients selected have a minimal L^1 norm [2]. BP differs from the Method of Frames only by the L^2 norm being replaced with the L^1 norm; however, this changes the form of the solution considerably. In BP, one solves the problem:

$$\min \|\alpha\|_1 \text{ subject to } \Phi\alpha = f \qquad (5)$$

where Φ is an $n \times p$ matrix of waveforms where $p > n$ (overcomplete dictionary) and α is the vector of coefficients. The MOF requires the solution of a quadratic optimization problem, and so the minimization is found in the first derivative where the minimum can be easily found. In contrast, Basis Pursuit requires the solution of a convex optimization problem with inequality constraints. Here it is necessary to use a conjugate gradient method to find the solution.

Because of the non-differentiability of the L^1 norm, BP leads to decompositions that can have very different properties from the Method of Frames. BP decompositions can be much sparser. Because Basis Pursuit always delivers a decomposition in an optimal basis and not necessarily an orthogonal basis, it seems better than the Best Orthogonal Basis method in resolving nonorthogonal structures; however the cost to achieve this is at the expense of greater computational complexity.

6 EXPERIMENTS

The ability of these methods for preserving the resolution in the reconstructed images with the wavelet packet dictionary for and MRI image, X-ray, and a photograph are observed. Initial results show MP with a compression ratio of 100:1 while the other methods show ratios from 16:1 to 30:1. From the figures, the reconstruction from MOF, BOB, and BP looks good to the naked eye. MP did not do well in these examples. This also shows that BP does not offer much of an improvement over MOF even with the added algorithmic complexity. MP does not perform well to reconstruct images as the other methods, but does yield superior compression ratios. The Peak-Signal-to-Noise Ratio (PSNR) is used to give a qualitative analysis of the images and their reconstruction [7]. The PSNR is given by:

$$PSNR = 10\log_{10}\left(\frac{255^2}{D}\right) \qquad (6)$$

D is the Mean Square Error (MSE), $E\left\{(x-y)^2\right\}$.

7 CONCLUSION

Investigation into applying dictionary methods to the problem of image compression has produced promising results. The characteristics of wavelets which include “compact support”, overcomes some of the limitations of image compression seen in traditional approaches. In contrast, traditional methods such as Fourier based dictionaries provide an effective means for representing signals that are smooth in nature or do not contain abrupt changes or variations; however, these types of dictionaries are not sufficient for representing a signal that may have many irregularities, possess unique features, or exhibits transient behavior [4]. As a result, wavelet dictionaries have been shown to perform well as or better than standard approaches. Table 1 and Chart 1 show the Peak Signal-to-Noise Ratio of the images in tabular and graphical form respectively for each method. The methods showing the best results, Basis pursuit and Best Orthogonal Basis are very close. Since the complexity of MOF is $O(n\log(n))$, BP is $O(n\sqrt{n}\log(n))$, MP is $quasi O(n\log(n))$, and BOB is $O(n\log(n))$, the complexity may become a factor in the selection of the best method.

	Method of Frames	Basis Pursuit	Matching Pursuit	Best Orthogonal Basis
Image A - mammogram	25.3324	49.8391	24.7532	48.1309
Image B - chest x-ray	24.4679	49.4032	18.2503	48.1329
Image C - neck x-ray	25.0605	49.3017	24.0115	48.8765
Image D - nonmedical image	24.0712	49.1011	22.4513	48.4441

Chart 1 - Peak Signal-to-Noise Ratios of four methods on four different images.

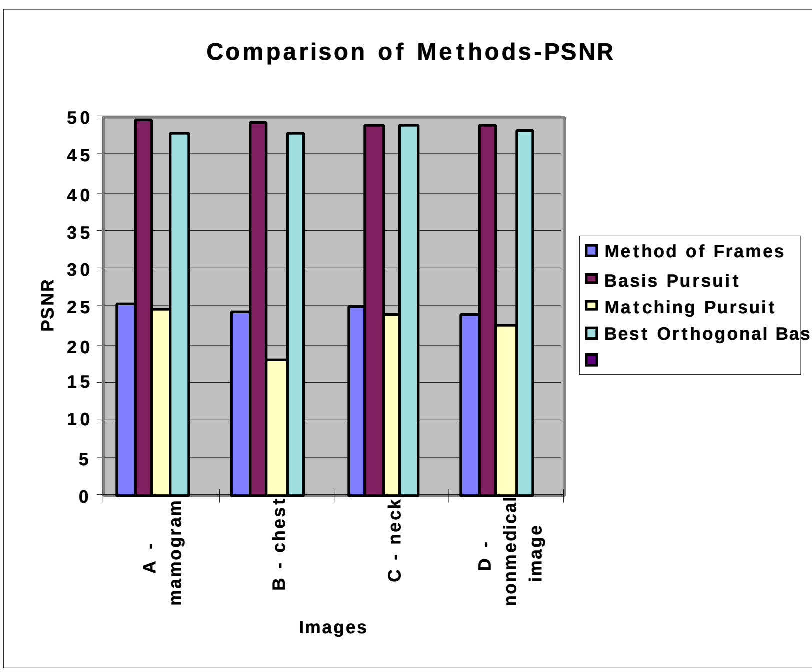

Table 1 - Data from Chart 1

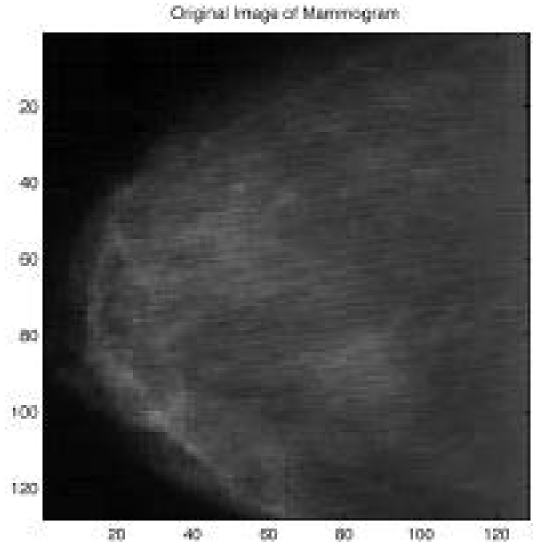

Figure 1- Image A-Original picture of breast.

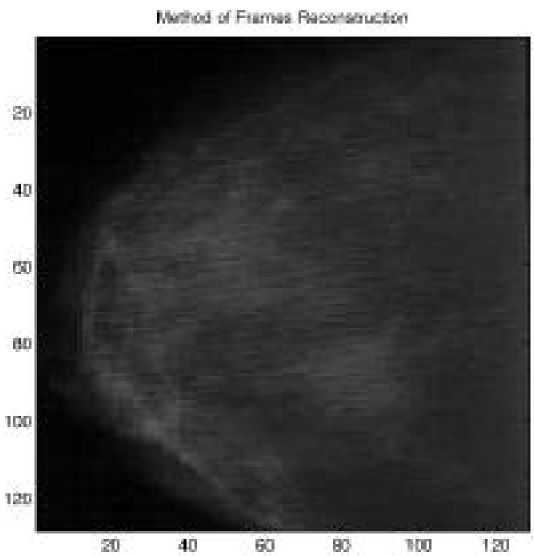

Figure 2 - Image A -Method of Frames reconstruction

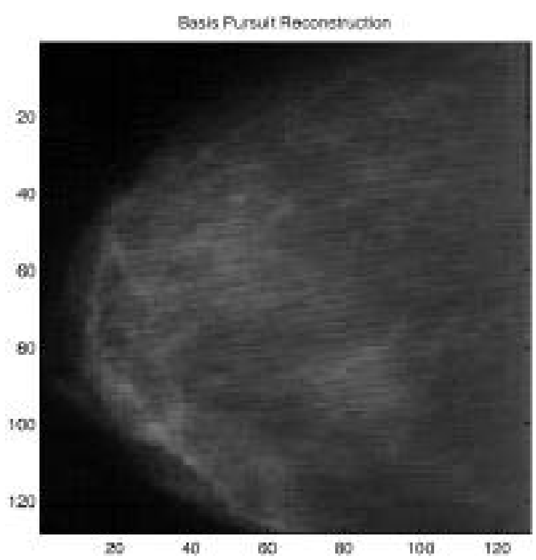

Figure 3 - Image A -Basis Pursuit Reconstruction

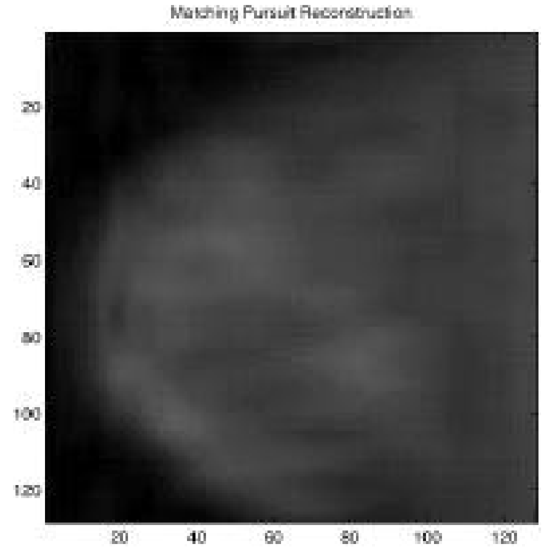

Figure 4 - Image A -Matching Pursuit Reconstruction

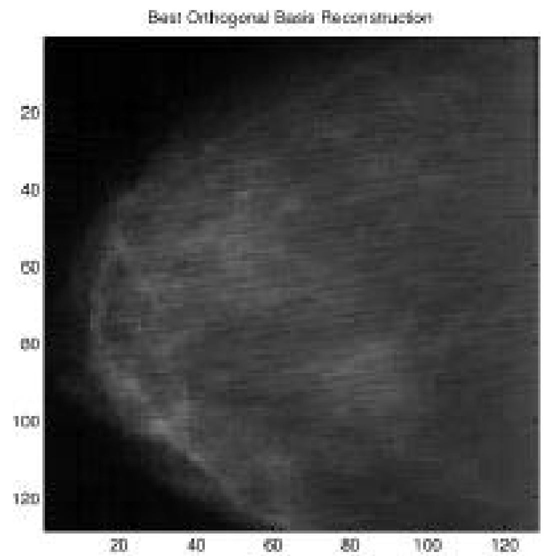

Figure 5 - Image A -Best Orthogonal Basis

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